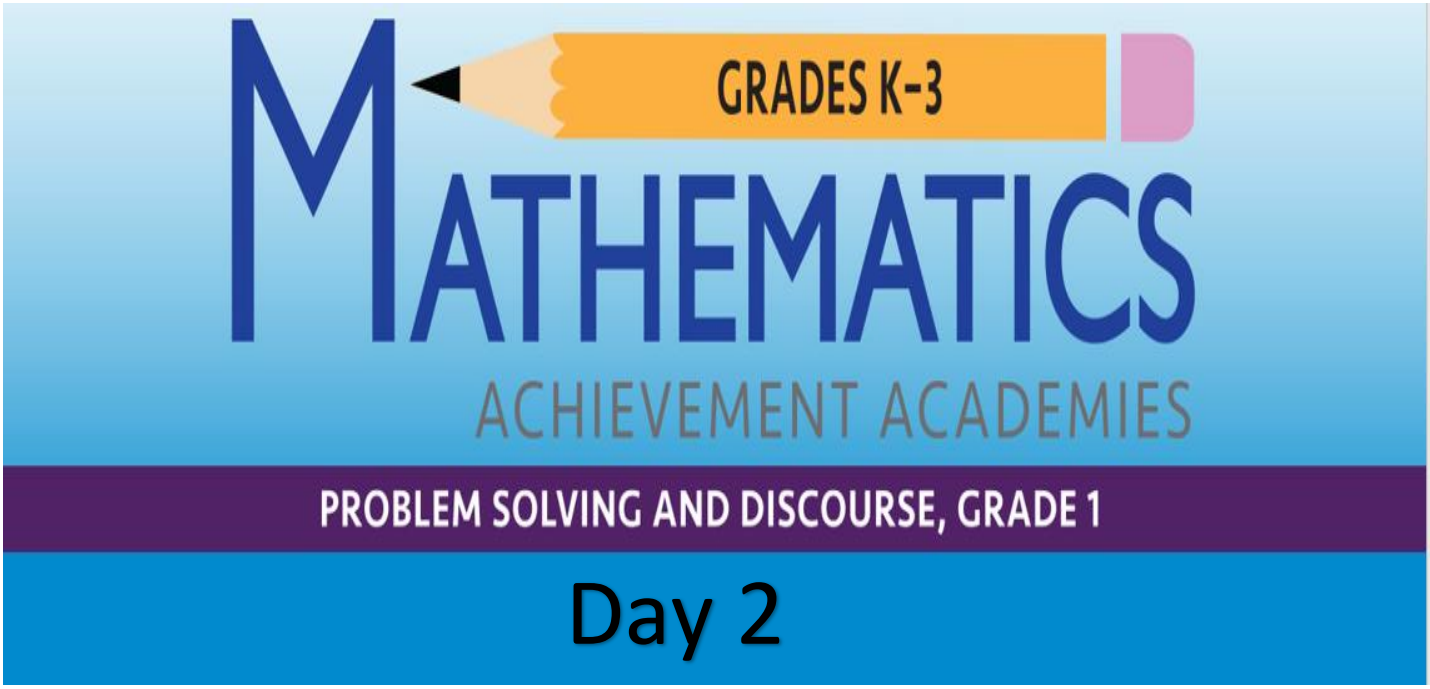
# Participant Journal - Virtual
# Texas Education Agency



## Day 1: Problem Solving and Student Voice



## Day 2: Problem-Solving Model and Equity

## DAY 2: PROBLEM-SOLVING MODEL AND EQUITY

### Mathematics Achievement Academy, Problem Solving and Discourse, Grade 1

### Outcomes

1. Build knowledge and understanding of a problem-solving heuristic that applies to any mathematical content strand.
2. Identify actions of a proficient problem solver.
3. Build knowledge and understanding of mathematical discussions that complement the use of a problem-solving heuristic.
4. Build knowledge and understanding of formative assessment strategies that focus on making student thinking visible to the teacher.
5. Build knowledge and understanding of five practices that support mathematical discourse to strengthen student voice and agency with a problem-solving heuristic.

### Day 2: Face-to-Face Professional Learning Session

*Learning Intentions*

1. Identify the actions of a proficient mathematics problem solver in order to support problem-solving development.
2. Define and understand the use of a problem-solving heuristic.
3. Understand appropriate applications of a problem-solving heuristic in grade 1 mathematics.
4. Explain each of the five practices for supporting mathematical discourse and related guiding questions.
5. Understand how a problem-solving model, five practices that support mathematical discourse, and mathematical language routines promote equity.

*Success Criteria*

1. I can describe the actions of proficient mathematical problem solvers and explain how to support the development of these characteristics.
2. I can explain the meaning of heuristic and describe how it applies to a problem-solving model.
3. I can describe appropriate applications of a problem-solving model heuristic in grade 1 mathematics.
4. I can explain each of the five practices and how they contribute to making meaning in mathematical discussions.
5. I can describe how a problem-solving model, the five practices that support mathematical discourse, and mathematical language routines promote equity.



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# Mathematics Achievement Academy: Problem Solving and Discourse, Grade 1 Academic Vocabulary Day 2

Equity
Exercise
Problem
Three-act tasks

Problem-solving model:

- Analyzing given information
- Formulating a plan
- Determining a solution
- Justifying the solution
- Evaluating the problem-solving process and the reasonableness of the solution

Mathematics TEKS 1(1)(B)

## Three-Act Tasks

Act 1:

- Provide the context for the inquiry through a picture or video.
- Prompt students to share what they notice about the picture or video.
- Prompt students to generate questions through wonderings.
- Identify the main question to be explored.

Act 2:

- Provide more information to help students answer the identified question.
- Facilitate conversations about reasonable answers.
- Allow students to explore and answer the question.

Act 3:

- Reveal the solution.
- Facilitate follow-up conversations as appropriate.

# Baseballs and Tennis Balls

Eduardo has 9 baseballs. He has 4 more tennis balls than baseballs. How many tennis balls does Eduardo have?

# Bean Bags

There were some bean bags in the gym. Five bean bags were lost. Now there are 14 bean bags. How many bean bags were in the gym at the start?

# Justifying the Solution and Evaluating the Problem-Solving Process

Teacher Actions and Student Actions

The teacher will

- prompt problem solvers to summarize big ideas
- provide scaffolds to help problem solvers use mathematical language to build their justifications
- prompt problem solvers to reflect on their own problem-solving process

so that the problem solvers will

- use their learning in future situations.
- become agents of their own mathematical and linguistic sense making.
- become more proficient in problem solving.

Promoting Sense Making in Problem Solving

Clarifying Student Understandings

Will you elaborate on why your solution answers the question?

Clarifying Understanding of Classmates

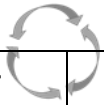
- Who can repeat_____'s thinking about the justifying the solution and evaluating the problem-solving process part of the problem-solving model?
- Will someone say what they just heard about_____'s justification?

Supports for Diverse Learners

# Problem-Solving Prompt

**Analyzing given information**

- What information do you know?
- What information do you need to know?
- How might problems you have seen like this one help you to understand this problem?
- How might you model this problem?



**Formulating a plan or strategy**

- What action does this problem make you think about?
- What strategies could you use to solve the problem?
- What is your plan?

**Determining a solution**

- Is your plan working? How do you know?
- Do you need to use a new plan or strategy?

**Evaluating the reasonableness of the solution**

- Does your solution answer the question?
- How do you know your solution makes sense?

**Justifying the solution**

How can this thinking help you solve more problems like this?

**Evaluating the problem-solving process**

How well did your plan work? Did you have to change your plan? Why?

# Proficient Problem Solvers

Analyze situations in mathematical terms.

Engage with problems willingly and persistently.

Consider simple cases of complex situations.

Recognize that some representations share common mathematical structures.
Categorize problems into types.

Use flexibility in thinking.

## Exercise Versus Problem

An **exercise** asks a student to practice a familiar skill.

Exercises allow students to

- practice a specific mathematical skill,
- demonstrate what was just learned,
- develop automaticity with a mathematical skill, and
- practice a specific mathematical skill in a context related to real-life experiences.

A **problem** involves the application of previously learned mathematical skills, concepts, and/or procedures to a situation where a solution process is not immediately apparent.

Problems allow students to

- apply a learned mathematical concept or skill where a solution is not obvious,
- demonstrate understanding of multiple mathematical concepts or ideas and their connections to each other,
- enter into a mathematical task with their current mathematical knowledge,
- build upon prior knowledge, and
- build new mathematical knowledge.

## Problem A

Gabby created a number sentence.

$$\square + \square + \square = 15$$

She used three different digits from this list:

1, 2, 4, 5, 6, 8, and 9.

What three numbers make the number sentence true?

Is there a different way?

## Problem B

10 students voted for their favorite fruit.

More than 3 students voted for apples.

Fewer than 4 students voted for bananas.

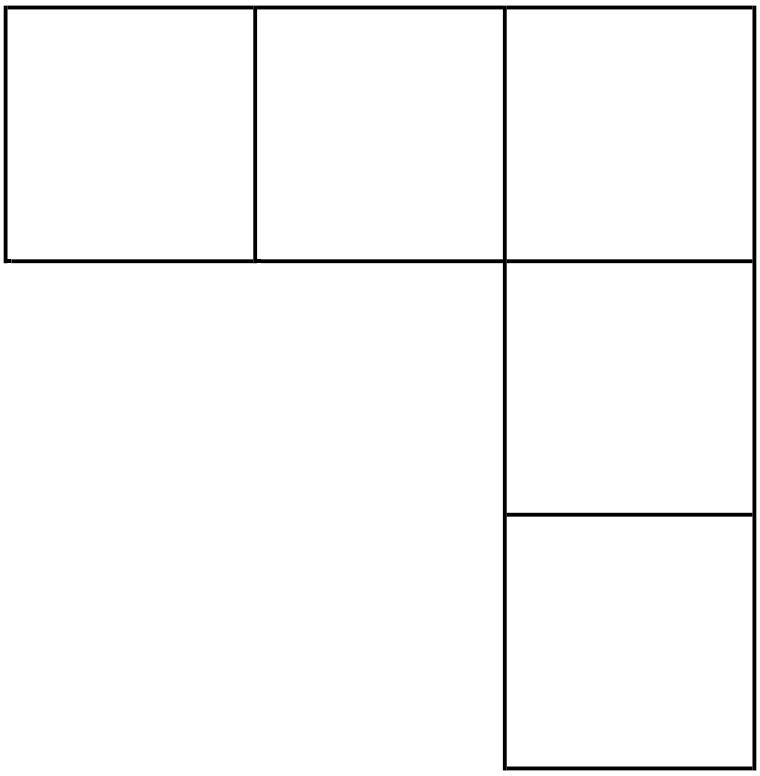
The rest of the students voted for oranges.

How many students voted for each fruit?

Is there a different way?

## Problem C

- The 3 numbers going across combine to make a sum.
- The 3 numbers going down combine to make a sum.
- The sums are the same.



Fill each square with one number: 1, 2, 3, 4, or 5.

Use each number only once.

Is there a different way?

## Problem D

There is a total of 13 seashells in the red, green, and blue pails.

The green pail has 1 more seashell than the red pail.

The red pail has 3 fewer seashells than the blue pail.

How many seashells are in each pail?

**Student A**

Can successfully compose and decompose addends to make jumps on the open number line

**Student B**

Uses basic drawings, such as circles, to represent counters used to determine solutions

**Student C**

Uses algorithms based on properties of operations to determine solutions

*For example, 3 + 8 + 6 may be added using the associative property as 3 + (7 + 1) + 6 = (3 + 7) + 1 + 6 = 10 + (1 + 6) = 10 + 7 = 17.*

**Student D**

Can successfully apply basic fact strategies, such as making 10, without the use of concrete or pictorial models

*For example, given the fact 7 + 8, a student may decompose 8 into 3 and 5 so that the 3 may be added to the 7 to make 10, then add the 10 and 5 to equal 15.*

# Flowers

Lam has a vase with some flowers.

- There are 9 red flowers.
- There are 5 white flowers.
- There are 4 yellow flowers.

How many flowers are in the vase?

Five Practices for Mathematical Discussions Cards

| The Five Practices | Descriptor |
|---|---|
| Sequencing | The teacher documents the order in which student work will be shared. |
| Connecting | The teacher plans questions to support students' thinking as they make comparisons between work samples. |
| Monitoring | The teacher listens and looks for mathematical thinking as students complete a task. |
| Anticipating | The teacher chooses a task and thinks about possible correct and incorrect student responses. |
| Selecting | The teacher chooses student work that highlights mathematical thinking that can move students forward. |

## Five Practices for Mathematical Discussions

Anticipating

- What approaches will students most likely use to solve the problem?
- What misconceptions may students demonstrate?
- What responses will support students while they are solving the problem?

Monitoring

- What mathematical ideas are being shown and discussed?
- What mathematical promise do I hear?
- What approaches are students demonstrating that were not anticipated?

Selecting

- Which student works reflect the anticipated approaches to solve the problem?
- Which student works include misconceptions or missteps that will be shared?
- Which student works show the needed mathematical pieces?

Stein, M. K., Engle, R. A., Smith, M. S., & Hughes, E. K. (2008). Orchestrating productive mathematical discussions: Five practices for helping teachers move beyond show and tell. *Mathematical Thinking and Learning, 10, 313–340*.

## Five Practices for Mathematical Discussions

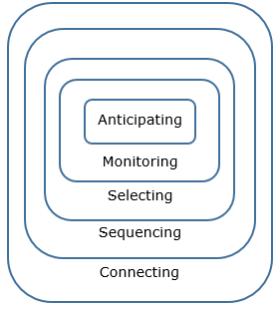
Sequencing

- Do I start with the most used strategy and then the lesser used strategy? Why?
- Do I start with the least complex strategy? Why?
- Do I want to use complementary or contrasting strategies? Why?
- In what order will students share their approaches to the problem?

Connecting

- What questions will make the mathematics visible to the students?
- What connections and relationships will students make based on the shared approaches?
- What discussions about accuracy and efficiency can we have?

Each practice is built on the practices embedded within it.




Stein, et al., 2008, 322

# Five Practices for Mathematical Discussions

**Anticipate:** What are three solution strategies that you think your students might use?

| | | |
|---|---|---|
| | | |

**Monitor/Selecting:** What three or four strategies do you want to highlight?

| | | | |
|---|---|---|---|
| | | | |
| 1 2 3 4 | 1 2 3 4 | 1 2 3 4 | 1 2 3 4 |

**Sequencing:** Circle numbers above to identify the order of the solution strategies/student thinking you want to highlight.

**Connecting:** What connection(s) do you want students to see?

| 1 and 2 | 2 and 3 | 3 and 4 |
|---|---|---|
| | | |

Circle the connections you expect to hear from students.

# Five Practices for Mathematical Discussions

**Anticipate:** What are three solution strategies that you think your students might use?

| *Pictures representing each quantity* | *Strategies*<br>• *make 10*<br>• *properties of operations* | *Counting-up on the open number line* |
|---|---|---|

**Monitor/Selecting:** What three or four strategies do you want to highlight?

| *Using pictorial models* | *Using making 10* | *Using properties of operations* | *Counting-up on the open number line* |
|---|---|---|---|
| *SF* *SB* | *SD* | *SE* *SB* | *SA* |
| (1) 2 3 4 | 1 2 (3) 4 | 1 2 3 (4) | 1 (2) 3 4 |

**Sequencing:** Circle numbers above to identify the order of the solution strategies/student thinking you want to highlight.

**Connecting:** What connection(s) do you want students to see?

| 1 and 2 | 2 and 3 | 3 and 4 |
|---|---|---|
| *representing quantities as pictures and using open number line* | *decomposing numbers on open number line and with numbers* | *using strategies such as make 10 and properties of operations* |

Circle the connections you expect to hear from students.

# Promoting Equity

| Equity-Based Practices[1] | Looks Like . . . | Reflection |
|---|---|---|
| Going deep with mathematics | Expect students to analyze, compare, justify, and prove their solutions while providing needed support. Provide tasks that are problems, allowing for multiple representations and solution strategies. | • How does my problem-solving instruction promote analysis?<br>• How do I support students in closely examining problem solving? |
| Leveraging multiple mathematical competencies | Identify and support mathematical contributions from all students. Present tasks with multiple entry points that allow students to make meaningful contributions to the group's learning. | • How do I identify and support problem-solving contributions from students with different strengths and levels of confidence?<br>• How do I encourage my students to embrace challenges while striving to improve daily? |
| Affirming mathematics learners' identities | Problem solve to promote reasoning and persistence. Learn from mistakes. Encourage students to see themselves as mathematicians. | • How do I structure my interactions with students to promote persistence with problems?<br>• How do I discourage my students from linking speed with math "smartness"? |
| Challenging spaces of marginality | Connect students' knowledge and experiences with mathematics. Increase opportunities for students to ask mathematical questions. Encourage participation by all students and student-to-student interactivity. | • How do I connect my students' knowledge (inside and outside the classroom) with the mathematics needed to solve problems?<br>• How do I make sure that all students have opportunities to demonstrate their problem-solving knowledge during the lesson? |
| Drawing on multiple resources of knowledge (math, culture, language, family, community) | Connect to prior mathematical learning. Learn students' knowledge and experiences. Learn from families and communities how to develop students as confident mathematicians. Communicate strengths and needs of students to families to promote learning. | • How do I get to know my students' backgrounds and experiences to support problem solving in my classroom?<br>• How do I affirm some of my students' multilingual abilities to help them become proficient problem solvers? |

[1]Aguirre, J., Mayfield-Ingram, K., & Martin, D.M. (2013). *The impact of identity in K-8 mathematics: Rethinking equity-based practices.* Reston, VA: The National Council of Teachers of Mathematics.

## Equitable Practices

| | Going deep with mathematics | Leveraging multiple mathematical competencies | Affirming mathematics learners' identities | Challenging spaces of marginality | Drawing on multiple resources of knowledge (math, culture, language, family, community) |
|---|---|---|---|---|---|
| Co-craft questions | | | | | |
| Compare and connect | | | | | |
| Discourse opportunities | | | | | |
| Three-act task | | | | | |
| Five practices for mathematical discussions | | | | | |
| Problem-solving model | | | | | |

## Equity-Based Practices for the Mathematics Classroom

| | |
|---|---|
| Going deep with mathematics | Expect students to analyze, compare, justify, and prove their solutions while providing needed support.<br>Provide tasks that are problems, allowing for multiple representations and solution strategies. |
| Leveraging multiple mathematical competencies | Identify and support mathematical contributions from all students.<br>Present tasks with multiple entry points that allow students to make meaningful contributions to the group's learning. |
| Affirming mathematics learners' identities | Problem solve to promote reasoning and persistence.<br>Learn from mistakes.<br>Encourage students to see themselves as mathematicians. |
| Challenging spaces of marginality | Connect students' knowledge and experiences with mathematics.<br>Increase opportunities for students to ask mathematical questions.<br>Encourage participation by all students and student-to-student interactivity. |
| Drawing on multiple resources of knowledge (math, culture, language, family, community) | Connect to prior mathematical learning.<br>Learn students' knowledge and experiences.<br>Learn from families and communities how to develop students as confident mathematicians.<br>Communicate strengths and needs of students to families to promote learning. |

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